Gamma-ray bursts and the relevance of rotation-induced neutrino sterilization

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Abstract. *À la* Pontecorvo when one defines electroweak flavour states of neutrinos as a linear superposition of mass eigenstates one ignores the associated spin. If, however, there is a significant rotation between the neutrino source, and the detector, a negative helicity state emitted by the former acquires a non-zero probability amplitude to be perceived as a positive helicity state by the latter. Both of these states are still in the left-Weyl sector of the Lorentz group. The electroweak interaction cross sections for such helicity-flipped states are suppressed by a factor of $(m_\nu/E_\nu)^2$, where m_ν is the expectation value of the neutrino mass, and E_ν is the associated energy. Thus, if the detecting process is based on electroweak interactions, and the neutrino source is a highly rotating object, the rotation-induced helicity flip becomes very significant in interpreting the data. The effect immediately generalizes to anti-neutrinos. Motivated by these observations we present a generalization of the Pontecorvo formalism and discuss its relevance in the context of recent data obtained by the IceCube neutrino telescope.

In models of GRBs (γ-ray bursts), ultrahigh energy neutrinos of several hundred TeV are expected to be emitted from accretion disk surrounding highly rotating black holes or neutron stars [1, 2, 3]. The emission in general is not isotropic. The IceCube neutrino detector has recently reported an absence of neutrinos associated with cosmic-ray acceleration in GRBs. The collaboration draws the consclusion that either GRBs are not the only source of cosmic rays with energies exceeding $10^{18}\,\mathrm{eV}$ or that efficiency of neutrino production is much lower, at least by a factor of 3.7, than has been predicted [4]. Several objections have been raised to this interpretation of the data [5, 6]. None of these works, however, incorporate the fundamental circumstance that the GRB neutrinos are produced in highly rotating frames while they are observed in a frame which may in comparison be considered as non-rotating. In conjunction with the observations contained in the Abstract above, this leads to a partial sterilization of the GRB neutrinos.

Motivated by these observation, we recall that in the standard neutrino-oscillation formalism *à la* Pontecorvo a flavour-eigenstate is a linear superposition of three mass eigenstates

$$|\nu_\ell, \sigma\rangle = \sum_{j=1,2,3} U^*_{\ell j} |m_j, \sigma\rangle, \quad \ell = e, \mu, \tau, \quad \sigma = -\frac{1}{2}. \tag{1}$$

Each of the underlying mass eigenstates corresponds to the same helicity, σ (at this stage $\sigma = +1/2$ is supressed by m_j/E). The 3×3 mixing matrix U is determined from experiments as are the mass-squared differences $\Delta m^2_{jj'} := m^2_j - m^2_{j'}$. For our purposes it suffices to assume that each of the mass eigenstates has four-momentum p_μ with $\mathbf{p}_j = \mathbf{p}_{j'}$. Thus flavour-oscillations, in this working framework, reside in different p_0 associated with each of the mass eigenstates.

With the recent IceCube null result in mind, we now consider a set up in which the source of neutrinos resides in a highly rotating astrophysical object, say a GRB. To calculate flavour oscillation probabilities for a neutrino-detector on Earth we recall that under a space-time translation $a^\mu = (t,)$, where represents the source-detector separation,

$$|m_j, \sigma\rangle \to e^{i p_\mu a^\mu} |m_j, \sigma\rangle. \tag{2}$$

and each of the mass eigenstate picks up a j-dependent phase factor. It is this j dependence that results in neutrino-flavour oscillations à la Pontecorvo [7, 8]. If in the frame of the observer, the source rotates at an angular frequency, $\boldsymbol{\omega} := \omega \hat{\mathbf{n}}$, $\hat{\mathbf{n}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, then each of the mass eigenstate undergoes a j-independent transformation

$$|m_j, \sigma\rangle \to \sum_{\sigma'} \left[\exp\left(i\frac{\boldsymbol{\sigma}}{2}\cdot\hat{\mathbf{n}}\,\omega t\right)\right]_{\sigma'\sigma} |m_j, \sigma'\rangle, \quad \sigma' \pm \frac{1}{2} \tag{3}$$

(where the change in momentum associated with the mass eigenstates is notationally suppressed). Because of the j-independence of this effect, the modified flavour-oscillation probability factorises

$$P(\ell, \sigma \to \ell', \sigma') = P(\sigma \to \sigma') P(\ell \to \ell') \tag{4}$$

In the above expression, $P(\ell \to \ell')$ is the usual flavour-oscillation probability of the standard formalism [9], while

$$P(\sigma \to \sigma') = \left[\exp\left(i\frac{\boldsymbol{\sigma}}{2} \cdot \hat{\mathbf{n}}\, \omega t\right)\right]^*_{\sigma'\sigma} \left[\exp\left(i\frac{\boldsymbol{\sigma}}{2} \cdot \hat{\mathbf{n}}\, \omega t\right)\right]_{\sigma'\sigma}, \quad \text{(no sum)} \tag{5}$$

Since $(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}})^2$ is a 2×2 identity matrix, $\mathbf{I}$, the exponential enclosed in the square brackets reduces to[1]

$$\cos\left(\frac{wL}{2}\right)\mathbf{I} + i\boldsymbol{\sigma} \cdot \hat{\mathbf{n}} \sin\left(\frac{wL}{2}\right). \tag{6}$$

A straight forward calculation then yields the modified expressions for flavour-oscillations

$$P\left(\ell, -\frac{1}{2} \to \ell', +\frac{1}{2}\right) = \sin^2\theta \sin^2\left(\frac{\omega L}{2}\right) P(\ell \to \ell'), \tag{7a}$$

$$P\left(\ell, -\frac{1}{2} \to \ell', -\frac{1}{2}\right) = \left[1 - \sin^2\theta \sin^2\left(\frac{\omega L}{2}\right)\right] P(\ell \to \ell'). \tag{7b}$$

These results are consistent with those found in the literature on magentic resonance [10]. For the isotropically-emitted neutrinos the standard averaging process over a sufficiently large patch of the sky gives

$$\left\langle P\left(\ell, -\frac{1}{2} \to \ell', +\frac{1}{2}\right)\right\rangle = \frac{1}{4}P(\ell \to \ell'), \quad \left\langle P\left(\ell, -\frac{1}{2} \to \ell', -\frac{1}{2}\right)\right\rangle = \frac{3}{4}P(\ell \to \ell'). \tag{8}$$

For models in which neutrinos are dominantly emitted perpendicular to the rotation axis one obtains[2]

$$\left\langle P\left(\ell, -\frac{1}{2} \to \ell', +\frac{1}{2}\right)\right\rangle = \frac{1}{2}P(\ell \to \ell'), \quad \left\langle P\left(\ell, -\frac{1}{2} \to \ell', -\frac{1}{2}\right)\right\rangle = \frac{1}{2}P(\ell \to \ell'). \tag{9}$$

Since the electroweak interaction cross section for the helicity-flipped states are suppressed by a factor of $(m_\nu/E_\nu)^2$, rotation acts to partially sterilize neutrinos. In consequence the interpretation of the data reported by IceCube suffers a modification and the expected neutrino events are reduced by the above-indicated factors (modulo the remark made in footnote 2). As a final remark we note that similar effects also arise via gravitationally-induced helicity transitions and these, together with the purely kinematical effect discussed here, show that the Pontecorvo formalism must be taken only as a first approximation in the neutrino oscillation phenomenology. Failure to do so can result in significant misinterpretation of the data.

[1] where we have set $t = L$ for ultrarelativistic neutrinos.

[2] If the dominant neutrino emission is along the axis of rotation, the resulting flavour-oscillation probability is roughly the same as in the Pontecorvo formalism.

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